

TEACHING TO SOLVE TASKS AT MATHEMATICS LESSONS: TEACHER'S INTELLECTUAL CHALLENGE

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Abstract

One of the methods to stimulate middle school students' cognitive activity is their teacher's intellectual challenge at the lesson. The results of American teachers' analysis have shown that the majority of them avoid challenge during lessons [1]. Avoidance of challenge is pedagogically contagious, meaning that a teacher avoiding challenge transmits this disposition to his students. Teacher's ability to pose a challenge while teaching mathematics is related to two factors: on the one hand, they are able to solve difficult problems during lessons and, on the other hand, they are capable of classifying tasks according to their level of difficulty. Two approaches have been used to reveal the term "difficult tasks" – these are the tasks requiring high cognitive demand and the tasks requiring high cognitive activity during their solution. Both approaches lead to similar classification of tasks: Tasks at Different Levels of Cognitive Demand [2] and Tasks at Different Levels of Achievement [3].

Intellectual challenge in solving difficult mathematics tasks during lessons is connected with implementation of teacher's creative and exploratory skills. To prove this thesis, 25 mathematics teachers from Kazan, Russian Federation, took part in the experiment. At the first stage, there was a questionnaire survey for teachers. Teachers' ability to understand the levels of cognitive demand in solving mathematical tasks was analyzed. Their capacity to model a list of tasks on one theme at different levels was researched. Next, there was the analysis of audio recordings of the lessons and the interviews with teachers. The results of this research have shown that even though the teachers have a high level of content and pedagogical knowledge they are not ready to accept challenge. According to theoretical criteria, only two teachers out of 25 were diagnosed to have posed a challenge to students. Other teachers' lessons demonstrated that their mode of behaviors, chosen teaching material and atmosphere at the lessons created poor conditions for its implementation. The reasons of this phenomenon are being analyzed. Methodological recommendations are being offered to provide mathematics teachers with psychological and methodological support while they are teaching students to solve mathematical tasks requiring intellectual challenge.

Keywords: cognitive demand, intellectual challenge, middle school mathematics teacher, mathematical task.

1 INTRODUCTION

The search of effective methodological techniques and methods to increase productivity of solving mathematical tasks by students is important and relevant for mathematics teachers. While solving mathematical tasks at the lesson a teacher and students should be bound by a single purpose – to develop students' ability to independently solve mathematical tasks. The teacher in this process should play the role of a "catalyst" who significantly increases the level of understanding how it is possible and necessary for students to tackle mathematical problems. Experience has shown that many mathematics teachers in solving the problem themselves often gracefully reform it, logically build the stages of its solution, appeal to beautiful mathematical ideas, but the student is not an active participant in the process of solving the problem. As H. Freudenthal noted, "the ideas of the solution fall through the sky, and are not the product of the student's reflections" [4].

For organizing the process of active stimulation of intellectual activity of pupils at mathematics lessons the teacher must possess a complex of psychological and professional qualities. A psychological portrait of a teacher is the following: he/she is in a positive, active, emotional and intellectual state stimulated by students' positive attitude to work [5]. Methodological features of a teacher's activity: he/she offers a novel training material that is unusual for students not only in content, but also in techniques and forms of learning activity (a new view on known facts, uncommon comparisons, questions with a different wording, changes in task data, advancement of unusual hypotheses of task solution, etc.). In this case we can speak about the readiness of a teacher to accept the intellectual challenge. The teacher challenging the students should react swiftly to their counter questions and

answers, getting a return intellectual challenge from the children, responding to his/her stimulating actions. Hence it can be affirmed that the concepts of “throw” and “accept” intellectual challenge are two sides of the same coin.

2 BACKGROUND

The most difficulty in teaching students to solve mathematical problems is development of research skills and improving motivation to solve problems (as to receive the result and to the process itself). Among other things, it depends on a teacher's ability to use cognitively demanding tasks in educational practice. Although the choice and the use of such tasks does not guarantee a high level of student's thinking [2; 6; 7], it ensures the development of research skills and a rise in motivation to solve mathematical problems by students having cognitive demand. A lot of researchers have been studying this problem. Boston, & Smith (2011) stated “Results indicate that teachers increased and sustained their ability to select high level instructional tasks and to maintain the level of cognitive demand during instruction. All teachers, however, did not exhibit this pattern. Portraits of teachers who continued to select and enact tasks at a high level are contrasted with those who did not, and factors are identified to account for teachers' current practices” [8]. The study of Choppin (2011) is devoted to the problems of adaptation of teachers to the use of complex mathematical tasks in teaching students. His results suggest that forming communities of inquiry around the use of challenging curriculum materials is important for providing opportunities for students to learn with understanding [9].

Teachers' ability to use cognitively demanding tasks in teaching practice has been studied by many researchers. According to Arbaugh & Brown (2005), “engaging teachers in learning to examine mathematical tasks using the LCD (Level of Cognitive Demand) criteria supports both a growth in pedagogical content knowledge (ways of thinking about mathematical tasks) and a change in practice (choosing mathematical tasks)” [10]. Studies by Monarrez and Tchoshanov (2013) have supported the idea to examine mathematical tasks using the levels of cognitive demand [11]. Their research is useful to clarify the interrelation between a teacher's knowledge and the use of tasks with a high level of cognitive demand in practice. Teachers with a strong background in content knowledge will be able to prepare students more effectively [12].

Stein and her colleagues (1990) differentiate mathematical tasks according to the level of cognitive demand of tasks (i.e. the level and the type of thinking, the development of which the problem causes) [2]. Analysis of the tasks has shown that the authors define two categories of mathematical tasks of high-level cognitive needs (“doing the math” and “procedures with connections”), and two categories of tasks with cognitive demand of a low level (“procedures without connections” and “on memorization”). As a result, the classification of educational mathematical tasks according to levels of difficulty in the process of cognition (Tasks at Different Levels of Cognitive Demand).

Close to the classification Tasks at Different Levels of Cognitive Demand is the classification by V.P. Bespalko who proposes to divide mathematical problems according to the level of acquisition as the ability to solve the problem [3]. The method of using acquired information in the process of solving problems has become the main principle of dividing activities of students into productive and reproductive. In reproductive activities, algorithms and rules are reproduced in various combinations – “from the literal copy and retelling to some free reproduction and use in typical situations, clearly defined by training, and a student in the course of activity does not add any new information to the initial data, acquired from a subject” [3]. In the process of productive activity, the student generates new information. Hence Bespalko divides tasks into the following levels: student level and algorithmic (based on reproductive activity), heuristic and creative (based on productive activities). For this classification, the author distinguishes in the problem the levels of certainty of the purpose, situations and actions in solving it (Table 1).

Table 1: Classification of levels of acquiring tasks by Bespalko.

Level of acquisition	Purpose	Situation	Actions to solve it
Student	Clearly defined	Clearly defined	Clearly defined
Algorithmic	Clearly defined	Clearly defined	Apply previously learned steps
Heuristic	Clearly defined	Not definite	Apply previously learned steps
Creative	Known in a general	Necessary to define	Necessary to define

This classification, in our opinion, allows to take into account both objective and subjective difficulties of students when solving problems. We use objective difficulties to mean the very structure of the task associated with the level of certainty of its components (Table 1), and subjective difficulties have been taken into account when determining levels of acquisition (development of the learner's experience), as well as in the formation of the components of each level (everyone has their own previously learned actions).

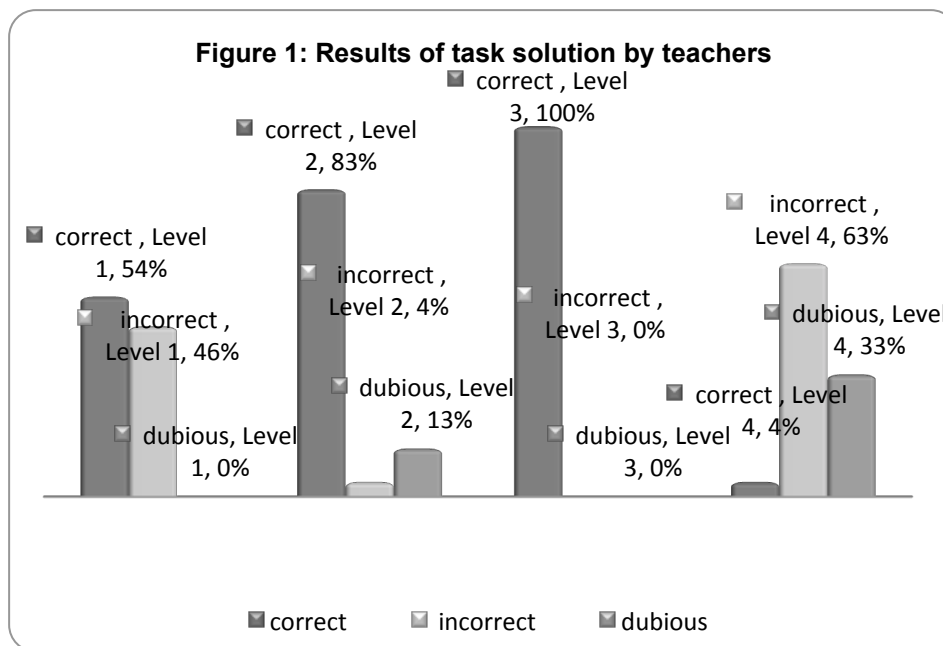
Teacher's understanding of an impact of a task of a certain level of difficulty on the development of a student's skills to solve mathematical problems allows the teacher to use a mathematical task timely and appropriately in the classroom. To make the task a stimulus for intellectual development of the student, it is necessary to aim it at enhancing productive cognitive activity of the student. The willingness of teachers to accept intellectual challenge was studied in the course of the experimental inspection by the American scientists from the University of Texas in El Paso (USA) under the supervision of Professor M.A. Tchoshanov. There was a survey that focused on high school mathematics teachers. The aim of it was to find out their disposition to intellectual challenge, in particular, to the solution of mathematical problems with an increasing level of complexity [13]. The correlation of this disposition with teaching practice and student performance was researched. The results of the study showed that most teachers refuse to accept intellectual challenge at lessons, and this avoidance of challenge is pedagogically contagious: a teacher avoiding challenge transmits this disposition to his students.

3 CONTENT, STRUCTURE AND BODY OF RESEARCH

The study involved 25 mathematics teachers from 5 schools of Kazan city of the Russian Federation. There was developed a survey program, including a questionnaire survey of secondary school mathematics teachers with subsequent analysis of their lessons and interview with them. The questionnaire consisted of 2 parts. In the first part of the questionnaire the teacher had to solve four tasks of various difficulty levels according to the following classification: the first level — tasks on reproduction (a question requiring reproduction of previously learned facts from memory (rules, formulas, definitions) and not requiring qualitative understanding); the second level — typical tasks (the simplest problems requiring application of known methods of solution in standard conditions); the third level — “atypical” tasks (tasks requiring qualitative understanding and implementation of mathematical methods of solution in non-standard conditions); the fourth level — research tasks (tasks requiring not only qualitative understanding of the training material, but also a creative use of it).

In the second part of the questionnaire the teacher had to offer their tasks of the four relevant levels of difficulty on the subject of “Area of triangle”. Classification and description of difficulty levels of mathematical problems were offered to the teachers in the questionnaire. Next, further qualitative analysis of audio recordings and written lesson plans on solving mathematical problems were conducted. This form of analysis of the teacher's activity to teach students to solve mathematical problems allows one to evaluate the following: the difficulty levels of the tasks at the lesson; the statement of questions in the process of solving the problem; the enthusiasm of the teacher; the proportion of independent work of students in the classroom, etc. The Last stage of the research was interviews with teachers.

The analysis of the suggested tasks solved by the teachers showed that 54% of teachers were able to solve the problem on reproduction of knowledge and only 4% managed the tasks of the research level (Fig. 1):



“Dubious” solutions are solutions that have some logical explanation based on correct mathematical concepts, but without a clearly proved solution. For example, a teacher finds the correct answer, writes platitudes about the solution but not the solution itself.

By applying the number of correctly solved tasks, we can conclude about the level of the teachers' preparation: only 37% of the respondents have a sufficient level of mathematical background (theory knowledge on the reproductive level, the ability to apply a theory to solving problems). The others make mistakes in definitions and are confused about their solutions.

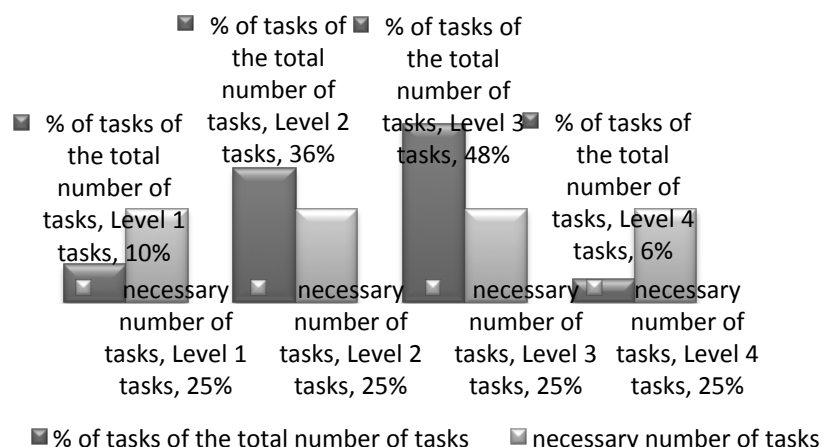
The teachers differentiated the solved tasks in accordance with the proposed classification. Task 1 of the second level of difficulty was correctly recognized by 80% of the teachers; task 2 of the first level of difficulty – 88% of the teachers; task 3 of the third level of difficulty – 80% of the teachers; task 4 of the fourth level – 70% of the teachers.

In the second part of the experiment, the teachers themselves offered the tasks of 4 difficulty levels on the subject of “Area of triangle”. We analyzed the proposed problems and their solutions. It turned out that no teacher was able to offer four tasks in ascending levels of difficulty. Only a third of the teachers suggested 3 tasks corresponding to difficulty levels. For example, the teacher offers the following tasks: the first level of difficulty: “What is the ratio of areas of two triangles if their heights are equal?”; the second level: “The area of a triangle is 30 cm². Find the height of a triangle if the base is 10 cm”; the third level: “The bisector BD divides the triangle ABC into two triangles. The area of the triangle BCD is 14 cm². Find the area of the triangle ABD, if AB = 8 cm, BC = 7 cm.” As the task of the fourth research level, the teacher introduces the task of the third level of difficulty.

Qualitative analysis of the suggested tasks by the teachers showed that according to the proposed classification the stated level of difficulty matches: 33% of the proposed tasks of the first level of difficulty; 62% of the tasks of the second level; 71% of the third level and 13% of the fourth level of difficulty.

Only 33% of the teachers were able to offer three out of four tasks in accordance with the proposed levels of difficulty. The greatest difficulty was caused by the problems of the research type; the teachers apparently do not understand the difference between a research problem and “atypical”. Next, the difficulty of selection of the tasks of the first simplest level appeared to be on the second place (33%). The conclusion is that the main difficulty for the teachers is to diagnose the problems of the 1st and the 4th levels of difficulty. This conclusion is confirmed by the following data. We will calculate the percentage of the tasks of each level of all teachers' proposed tasks (Fig. 2). Each level should take 25% of the total number of the proposed tasks by the teachers. But almost half (48%) of the proposed tasks turned out to be the atypical tasks of the third level. Analysis of the questionnaires showed that most of the tasks of the 3rd level were substituted by the research tasks of the fourth level. The tasks of the first level were often substituted by the tasks of the second level.

Figure 2: Number of tasks of different levels of difficulty offered by teacher, of the total number of tasks



Based on the results of the questionnaire of the teachers, we can conclude that only 4 teachers out of 25 are ready to implement the diagnosis of the tasks and invent such tasks. This conclusion was obtained by generalizing the results of the first and second stages of the survey.

The next stage of the research consisted in the analysis of the teachers' lessons in order to examine their disposition to intellectual challenge. For this purpose, every teacher provided their audio lesson of problem solution and attached their lists of tasks and lesson plans. Thus in the process of solving problems in the classroom the most comfortable conditions were created for the teacher (an arbitrary topic, no video shooting, an opportunity to record a lesson several times and choose the best, etc.). It was noticed that after the questionnaire the teachers tried to improve the selection of tasks (to make it diverse according to the level of difficulty, add research tasks). The following criteria were taken into consideration while analyzing the tasks in the classroom: the number of tasks of various levels of difficulty at the lesson, the teacher's leading questions; the teacher's actions when errors occur in the solution of the problem; the behavior of the teacher. Based on these criteria, it was being defined whether intellectual challenge in solving mathematical problems had been achieved. It was common that all teachers did not have tasks of a research type at their lessons. Even the teachers who had higher results in the questionnaire mainly chose tasks of the second and third levels of difficulty in class. Based only on this criterion, we can conclude that no teacher implemented intellectual challenge in the process of solving mathematical problems. We will note some of the methods of the teachers' actions which did not contribute to the organization of intellectual challenge in solving mathematical problems: the teacher's tips were in the form of recommendations to use specific theorems and definitions; the teacher begins a familiar phrase requiring continuation; the teacher never listens to students' proposals for solving a problem, because they are not relevant to his/her solutions; easy tasks were chosen not causing any difficulties for students, etc.

Among teachers who are able to "throw" intellectual challenge to students, we can distinguish two groups. These are the characteristics of the teachers of the first group: they solve the problems of the 1st, 2nd, 3rd levels of difficulty (and even some research tasks) well, have extensive experience (as a rule, they are over 40 years old), self-confident (confident speech, calm confident communication with students), but do not have high expectations from the students in solving the proposed tasks. They do not give students time to think over the solution of the problem by themselves, begin to suggest necessary theorems, definitions, facts in case of the slightest difficulty, actively guide students towards the solution of the problem, etc. Gradual implementation of the lesson plan is observed, but it is not the activation of creative and research potential of students when solving problems. The features of the second category of the teachers: they solve the problems of the 1st, 2nd, 3rd levels of difficulties well, have an idea about the research objectives, reflect on methodological issues (e.g., they build a system of objectives in a beautiful and methodically competent way), are respectful towards their students' solutions (listen carefully, make conclusions based on the results of task solution); understand the depth of the intellectual potential of students, but are not fluent in solving research-level problems, so avoid them or offer very rarely.

While interviewing the teachers of both groups the following can be noted. The teachers of the 1st group have a very strong position in the organization of the lesson, they are interested in new mathematical problems and approaches to learning that do not contradict their attitudes. The teachers of the 2nd group would like to “throw” intellectual challenge to students. They understand that it is necessary for the development of intellectual abilities of their students, for upbringing of courage and determination when they are facing problems and challenges requiring non-standard approach for their solution. The teachers feel particularly anxious when dealing with gifted children who themselves pose intellectual challenge to the teacher.

4 DISCUSSIONS AND CONCLUSION

For the productivity of mathematical education of pupils it is central for secondary school teachers to understand the importance of such pedagogic categories as cognitive demand. The study by Monarrez, & Tchoshanov (2013) showed that teachers of mathematics in the USA are able to recognize the levels of cognitive needs and challenges of the presented task [11]. Our study has shown similar results: on average, 80% of the teachers who participated in the experiment determine the difficulty levels of tasks correctly. Like their American colleagues, they face difficulties in designing tasks of different levels of cognitive demand, particularly the tasks which are focused on procedures without connections and requiring connections between procedures.

Researchers Hill, Rowan, and Ball (2005) studied the effects of teachers' mathematical knowledge on student achievement. They found that teachers' mathematical knowledge was significantly related to student achievement gains [14]. If the mathematics teacher provides students only with tasks on memorization and algorithm, it restricts the development of their thinking [15]. In addition, the teacher's knowledge and the student's knowledge are parallel, that has been documented [16]. That means that the teacher's disposition to the solution of complex mathematical tasks and problems is transmitted or develops the tendency of the students facing such tasks not to give up and accept the challenge. The teacher's lack of sound knowledge, the fear and anxiety when solving complex mathematical problems directly affect the students, they are not ready to accept intellectual challenge and tackle difficult tasks since the teacher has not proposed these tasks to them [17; 18].

It is established that the teacher's view about the level of difficulty of a mathematical problem is connected with his/her level of mathematical preparation: the teacher solving mathematical problems well classify them better. The greatest difficulty was caused by the problems of a research level both in the solution of the proposed tasks (part 1 of the questionnaire) and in the selection of such tasks (part 2 of the questionnaire). Research tasks are the foundation for the organization of intellectual challenge at mathematics lessons. Only 4 out of 25 teachers were able to correctly offer the task of a research level, but only two of them used these tasks in the classroom.

The results of this study have shown that intellectual challenge in solving mathematical problems is a rare phenomenon at mathematics lessons at school. The reasons for this are both objective and subjective. Objective reasons lie in current traditional approaches to the methodology of solving mathematical problems, problem statements offered in textbooks, nature and number of connections, design of the text. Analysis of Russian mathematics textbooks of grades 7-9 has showed that there are practically no tasks of the first and fourth levels of difficulty and problems do not always reveal the concept under consideration at reproductive and productive levels [19]. Subjective reasons lie in the personal experience of mathematics teachers and their established professional competencies [20].

In the process of research, it has been established that there are teachers who have potential opportunity to use intellectual challenge in solving problems in the classroom. They have a high level of professional training. However, they do not feel the need to use this technique when solving mathematical problems. According to the results of the survey, there are teachers who want to “throw” intellectual challenge to students, but they do not know how to do it. They are afraid to lose credibility with children if intellectual challenge in the classroom is not successful. So they are limited to the use of research tasks as homework.

To provide psychological and instructional support to mathematics teachers it is required to work purposefully on the design of learning technology for solving mathematical problems requiring acceptance of intellectual challenge. An important problem is to evaluate the teaching of solving mathematical problems in the process of intellectual challenge. It requires a special study.

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